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LETTER TO THE EDITOR

New series expansion data for surface and bulk resistivity and conductivity in two-dimensional percolation

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Abstract. Low-density series expansions are obtained for random resistor networks. The expansions are obtained on the square lattice to order 18 and on the triangular lattice to order 14. Bulk and surface expansions are given for both resistive and conductive susceptibilities. The balance of the evidence obtained from analysing these series is in favour of the value $\zeta_R = 1.32 \pm 0.02$ for the critical exponent of the resistance scale and supports the existence of only a single such scale for bulk and surface susceptibilities. This value is in agreement with earlier Monte Carlo work.

Previous series expansion work on two-dimensional random resistor networks was for bond percolation on the square lattice. Fisch and Harris [1] obtained the low-density expansion of the resistive and conductive susceptiblities to order p^{10} and this was subsequently extended to order p^{16} by Essam and Bhatti [2]. The resistive susceptibility is defined by

$$\chi_R(p) = \sum_{\boldsymbol{r} \neq \boldsymbol{0}} R(\boldsymbol{r}, p) \tag{1}$$

where each lattice bond has probability p of being a unit conductor and $1 - p$ of being an insulator. $R(\boldsymbol{r}, p)$ is the expected resistance between some lattice site taken as the origin and a site with position vector $\boldsymbol{r}$, given that both sites belong to the same finite cluster. The conductive susceptibility, $\chi_C(p)$, is similarly defined with the resistance, $R(\boldsymbol{r}, p)$, replaced by its reciprocal. Here we extend the square lattice results to order p^{18} and obtain a completely new expansion for the triangular lattice to order p^{14}. Expansions are also obtained for the resistive and conductive susceptibilties, $\chi_{R1}(p)$ and $\chi_{C1}(p)$, of the corresponding semi-infinite lattices when the origin is chosen to be a point in the surface. The method used is described in [3] for the bulk and is easily extended to the surface problem. The calculation, including the generation of the non-nodal graph list, was completely automated and the number of terms obtained was limited only by availabilty of computer power. The series coefficients are given in the appendix.

Replacing $R(\boldsymbol{r}, p)$ by $P(\boldsymbol{r}, p)$, the probability of a conducting path to $\boldsymbol{r}$, in (1) gives the mean cluster size functions $S(p)$ and $S_1(p)$, which are the expected number of sites connected to the origin in the bulk and semi-infinite lattices, respectively. On approaching

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the critical probablity p_c, the bulk cluster size and resistive susceptibilities are normally assumed to diverge with critical exponents γ and γ_R with corresponding surface exponents γ_1 and γ_{R1}. The values of p_c are known for the two lattices considered: $p_c(\text{square}) = \frac{1}{2}$ and $p_c(\text{triangular}) = 2\sin(\pi/18)$. Normalizing the susceptibilities by dividing by the corresponding mean size gives the resistance and conductance scales denoted by $L_R(p)$, $L_C(p)$, $L_{R1}(p)$ and $L_{C1}(p)$. Thus the resistance scale is

$$L_R(p) \equiv \frac{\chi_R(p)}{S(p)} \sim (p_c - p)^{-\zeta_R} \tag{2}$$

which diverges with exponent $\zeta_R = \gamma_R - \gamma$. The conductance scale, $L_C(p)$, tends to zero at the critical point and has negative exponent $\zeta_C = \gamma_C - \gamma$ which is normally assumed to have magnitude ζ_R [1], this being in agreement with our results below. We also find that the scales for the surface problem have the same exponents as for the bulk properties. This is to be expected since there can be no surface transition and the only scales which become singular at the transition are those of the bulk (i.e. there is only one connectedness length exponent ν at an 'ordinary' transition). Assuming the above relations between the scale exponents, we are able to obtain four separate estimates of ζ_R for each of the two lattices.

Bhatti and Essam [3] analysed their 16 term $\chi_R(p)$ series for the square lattice by the methods of Baker and Hunter (BH) [4] and the M2 method of Adler *et al* [5, 6] and found $\zeta_R = 1.26 \pm 0.02$ which supported the Alexander–Orbach (AO) conjecture [7] $\zeta_R = \frac{1}{2}\Delta$, where Δ is the critical exponent for the scale of the cluster size distribution. (In two dimensions $\Delta = \frac{91}{36}$ [8] which gives $\zeta_R = 1.263\,8\dot{8}$.) The large error in their estimate $\zeta_C = -1.3 \pm 0.1$ reflected the relatively poor convergence of the χ_C series.

The result of den Nijs [8] that $\gamma = 2\frac{7}{18}$ is normally accepted as exact and when this was first discovered there was an inconsistency with some series expansion estimates based on DLog Padé approximants. This was resolved by Adler *et al* [6] who pointed out the importance of allowing for corrections to scaling in the analysis of percolation series. Thus the scaling form of $\chi_R(p)$ is just the leading term in an asymptotic expansion:

$$\chi_R(p) \cong A(p_c - p)^{-\gamma_R}\left(1 + \sum_{i=1}^{\infty} B_i (p_c - p)^{\Delta_i}\right). \tag{3}$$

The presence of such correction terms slows down the convergence of the usual DLog Padé approximant technique and the BH and M2 methods combat this by first transforming the expansion of $\chi_R(p)$. This improves the accuracy of the leading exponent γ_R and the leading correction exponent Δ_1 can often be estimated. The value obtained for Δ_1 may only be an effective value since it can be strongly influenced by higher order corrections. If it were the true value then Adler's assumption [9] that its value is the same for the mean size and both susceptibility series would be reasonable. Making this assumption she found, by reanalysing the series of Fisch and Harris [1], that $\zeta_R = 1.31 \pm 0.20$ where the large tolerance arises by allowing values of Δ_1 in the range 1.1–1.4.

The result of Essam and Bhatti [2] is in conflict with several accurate computer simulations [10–13] which rejected the Alexander–Orbach conjecture. In particular, Normand *et al* have estimated ζ_R using extensive Monte Carlo simulations of both bond and site percolation on a special purpose computer [14]. In their initial analysis, they estimate the leading order exponent from a log–log plot for both site and bond percolation. Although the site and bond lines do not become parallel, as expected in the asymptotic limit, Normand *et al* were able to assign upper and lower bounds, $1.28 > \zeta_R > 1.31$. These bounds would exclude both the Alexander–Orbach conjecture and the alternative conjecture

$\zeta_R = \nu = \frac{4}{3}$ [15]. To improve the precision of their estimate, Normand *et al* also included a non-analytic correction to the scaling term in the analysis and chose the value of the leading exponent and correction exponent which appeared to give the best approach to the expected asymptotic behaviour for both the bond and site percolation results. This led to an estimate of $\zeta_R = 1.299 \pm 0.002$. In interpreting the increased precision of the estimate obtained in this way, it must be remembered that universality of the correction to the scaling exponent is insisted on; however, here again this exponent may be regarded as an effective exponent both because higher order correction terms are not included and because the analysis is effectively a nonlinear fit to four unknowns for each set of data.

Adler *et al* [5] considered the *divided* series $\chi_R(p) \div \chi_C(p)$ in the hope that the unknown non-analytic corrections to scaling would be swamped by the induced analytic corrections. (The divided series $A(p) \div B(p)$ is the series having coefficients which are the ratios of those in the expansions of $A(p)$ and $B(p)$.) In this way they obtained values of ζ_R ranging between 1.23 and 1.29. The other possiblity they considered was the presence of logarithmic corrections. The choice of exponent for the logarithm which made $\zeta_C = -\zeta_R$ gave $\zeta_R = 1.31 \pm 0.02$.

In the analysis of our new data we have applied three different methods to each series, the BH and M2 methods and also the method M1 described by Adler *et al* [5]. The last method treats the leading exponent as an independent variable and determines the value which minimizes the dispersion of the Δ_1 estimates. For each lattice and susceptiblity type we have considered four different series. Bhatti and Essam originally analysed $\chi_R(p)/p$ since the power expansion had no constant term. Here we find that introducing a constant term by using $1 + \chi_R(p)$ can make a significant difference to the critical exponent estimate in some cases. In particular the BH estimate of $\gamma_R - \gamma$ changes from 1.27 ± 0.05, which agreed with the AO conjecture in [3], to 1.31 ± 0.04 which is in line with the Monte Carlo estimates. In both cases the exact value of γ [8] (γ_1 [16]) is subtracted from the estimate of γ_R (γ_{R1}). We have also used the series for the derivative and the 'divided series' defined above, the latter gives gives a direct estimate of ζ_R. The results are listed in tables 1–4.

Although all three methods for a given expansion normally give estimates of Δ_1 which are consistent with one another, it can be seen that there is a wide variation in the value of

Table 1. Estimates of ζ_R from bulk resistive susceptibility series.

		Square		Triangular	
		$\gamma_R - \gamma$	Δ_1	$\gamma_R - \gamma$	Δ_1
$1 + \chi_R$	M2	1.31±0.02	1.75±0.01	1.35±0.02	1.6±0.1
	M1	1.32±0.01	2.1±0.1	1.35±0.01	1.6±0.2
	BH	1.31±0.04	1.7±0.1	1.30±0.05	1.2±0.5
χ_R/p	M2	1.31±0.02	0.9±0.1	1.29±0.03	1±0.2
	M1	1.30±0.03	0.9±0.3	1.31±0.05	0.6±0.4
	BH	1.27±0.05	1.3±0.2	1.29±0.04	1.2±0.4
$d\chi_R/dp$	M2	1.31±0.01	1.7±0.2	1.34±0.02	1.5±0.2
	M1	1.32±0.01	2.4±0.7	1.35±0.02	1.8±0.2
	BH	1.32±0.02	1.7±0.1	1.32±0.05	1.3±0.1
$\chi_R \div S$	M2	1.27±0.05	1.1±0.1	1.33±0.01	0.7±0.1
	M1	1.3±0.1	1.0±0.5	1.32±0.01	0.75±0.10
	BH	1.34±0.13	0.68±0.01	1.32±0.10	0.7±0.1

Table 2. Estimates of ζ_R from surface resistive susceptibility series.

		Square		Triangular	
		$\gamma_{R1}-\gamma_1$	Δ_1	$\gamma_{R1}-\gamma_1$	Δ_1
$1+\chi_{R1}$	M2	1.26±0.01	1.9±0.2	1.315±0.005	1.74±0.01
	M1	1.27±0.02	†	1.32±0.01	2.0±0.2
	BH	1.28±0.02	*	1.34±0.05	1.6±0.1
χ_{R1}/p	M2	1.24±0.18	1.7±1.3	1.29±0.02	1.0±0.2
	M1	1.26±0.03	1.1±0.1	1.29±0.02	1.0±0.2
	BH	1.29±0.05	1.1±0.1	1.27±0.08	1.0±0.3
$\mathrm{d}\chi_{R1}/\mathrm{d}p$	M2	1.27±0.01	2.4±0.1	1.31±0.01	1.62±0.02
	M1	1.265±0.005	2.4±0.4	1.31±0.01	1.65±0.07
	BH	1.28±0.02	2.6±0.2	1.33±0.03	1.7±0.2
$\chi_{R1}\div S_1$	M2	1.3±0.1	0.7±0.3	1.30±0.02	0.9±0.1
	M1	‡	‡	1.33±0.02	0.8±0.1
	BH	1.3±0.1	0.7±0.1	1.30±0.06	0.68±0.03

Table 3. Estimates of ζ_R from bulk conductive susceptibility series.

		Square		Triangular	
		$\gamma-\gamma_C$	Δ_1	$\gamma-\gamma_C$	Δ_1
$1+\chi_C$	M2	1.37±0.02	2.2±0.3	1.36±0.02	1.5±0.1
	M1	1.38±0.01	†	1.353±0.004	2.1±0.1
	BH	1.33±0.07	*	1.29±0.06	*
χ_C/p	M2	1.37±0.08	1.5±0.5	1.36±0.05	1.8±0.7
	M1	‡	‡	1.34±0.15	0.6±0.3
	BH	1.31±0.09	*	1.33±0.08	*
$\mathrm{d}\chi_C/\mathrm{d}p$	M2	1.37±0.08	1.5±0.5	1.37±0.06	2.0±1.0
	M1	1.43±0.04	†	1.41±0.03	†
	BH	1.31±0.09	*	1.31±0.03	*
$\chi_C\div S$	M2	1.27±0.02	1.4±0.1	1.27±0.01	1.6±0.1
	M1	1.27±0.01	1.4±0.1	1.25±0.01	1.6±0.3
	BH	1.24±0.03	1.6±0.5	1.25±0.02	1.7±0.5

Δ_1 from series to series. In most cases this exponent either has a value close to 1 (analytic correction) or close to 1.7. A notable exception to this is for the divided series of the resistive susceptiblity where $\Delta_1 < 1$. We therefore suppose that our results only estimate an effective correction to scaling.

In the tables an asterisk by the Baker–Hunter estimate means that no satisfactory estimate of Δ_1 could be obtained due to the widespread appearance of defective approximants. A dagger in an M2 row indicates that the estimate was based on the position of a discontinuity in the Δ_1 against γ curve which makes the assignment of a precise value to Δ_1 impossible. Such discontinuities occur naturally in test series and the evidence from such series is that the correct value of γ is indicated. A double dagger in an M2 row indicates that there was a continuous variation of γ with Δ_1 with no obvious converged region so that neither exponent is estimated.

According to universality and scaling, all of the leading exponent estimates in the tables

Table 4. Estimates of ζ_R from surface conductive susceptibility series.

		Square		Triangular	
		$\gamma_1 - \gamma_{C1}$	Δ_1	$\gamma_1 - \gamma_{C1}$	Δ_1
$1 + \chi_{C1}$	M2	1.40±0.01	1.7±0.2	1.39±0.02	1.41±0.02
	M1	1.38±0.01	†	1.382±0.002	1.7±0.1
	BH	*	*	1.32±0.06	*
χ_{C1}/p	M2	1.388±0.005	2.0±1.0	1.368±0.003	1.55±0.05
	M1	1.39±0.01	†	1.378±0.01	1.3±0.2
	BH	*	*	1.29±0.06	*
$d\chi_{C1}/dp$	M2	1.4±0.1	2.3±0.7	1.37±0.04	0.1±0.4
	M1	‡	‡	‡	‡
	BH	*	*	1.34±0.05	*
$\chi_{C1} \div S_1$	M2	1.24±0.02	1.7±0.1	1.25±0.01	1.75±0.05
	M1	1.25±0.01	1.6±0.1	1.24±0.01	1.9±0.2
	BH	1.23±0.03	1.8±0.2	1.245±0.015	1.7±1.0

should be the same and equal to ζ_R. The bulk resistive susceptibility estimates for both the square and triangular lattices (table 1) are consistent with 1.32 ± 0.02 in agreement with universality and with the Monte Carlo estimates referred to above. This result also covers the surface resistive susceptibility of the triangular lattice (table 2) in agreement with there being a single resistive scaling length for bulk and surface properties. However, the corresponding results for the square lattice are in better agreement with the AO conjecture (but violating universality) and there are in fact some very well converged results which support this hypothesis. However, assuming universality, the balance of the evidence favours the higher Monte Carlo result.

It is possible to *refine* the consistency of the estimates somewhat by insisting that certain conditions which are true in the asymptotic limit be met. For example, we may insist that both the leading exponent and the coefficient of the leading order term are the same when obtained from both χ_R/p and $1 + \chi_R$. If the estimates of coefficient against exponent obtained from each Padé approximant in the Baker–Hunter analysis for the two series are plotted on the same graph the crossing point provides an estimate which satisfies the condition. The difficulty with such approaches is similar to the difficulty with insisting on the universality of the correction to the scaling exponent, as was done in some of the earlier studies described above; that is, the coefficient has to be regarded as only an effective value for any analysis based on a finite number of terms. Consequently, any increased precision in the estimates obtained for a given pair of series may be an artefact of the condition imposed. It is therefore preferable to consider the full range of the central estimates of the exponent obtained for the various series and methods of analysis shown in the table and to take the variation in these central estimates as a reasonable indicator of the accuracy of the results.

Turning now to the conductive susceptibility, we notice a marked deterioration in the quality of the data. In particular, it is very often impossible to give an effective correction to the scaling exponent which suggests that our assumed form (3) is not a good representation of the conductive susceptibility functions. The resulting larger error bars attached to these exponents usually allow consistency with the scaling hypothesis $\gamma_R - \gamma = \gamma - \gamma_C$ [4] although there are some well converged exceptions. In particular, if one wanted to make a case for believing the AO conjecture then the data for $\chi_C \div S$ and $\chi_{C1} \div S_1$ would be excellent ammunition. However, again, our data suggest that the spread in the central estimates obtained by applying different methods of analysis and using different series with the same expected universality class, provides a more reliable error estimate. From this point of view, the conductivity series results cannot altogether rule out the AO conjecture in two dimensions; however, the overall central estimate would be consistent with the estimate from the resistivity series quoted above and would be substantially higher than the value given by the AO conjecture.

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Appendix

Table A1.

	Triangular lattice: bulk	
	Resistive susceptibility	Conductive susceptibility
0	0	0
1	6	6
2	60	15
3	386	46
4	2038	132.1
5	9616	376.1
6	42 020.363 636 363 636 363 636 363 636 36	1079.232 034 632 034 632 034 632 034 632
7	172 537.545 454 545 454 545 454 545 454 5	3097.891 466 325 212 455 243 414 995 726
8	682 760.923 778 282 142 753 158 019 519 1	8913.576 026 693 624 390 830 915 901 259
9	2 604 618.536 239 649 465 759 358 785 352	25 697.171 637 807 217 258 920 840 843 41
10	9 658 838.965 504 983 532 940 507 581 116	73 965.880 221 763 578 554 180 259 664 03
11	35 212 529.248 234 918 829 012 620 530 11	213 942.709 809 992 182 742 836 888 085 8
12	125 117 538.373 942 899 375 189 787 349 8	615 702.338 452 045 513 856 868 833 755 3
13	440 267 268.722 558 972 004 250 647 532 9	1 780 763.077 500 282 509 726 594 390 051
14	1 523 838 898.324 470 495 546 761 050 699	5 141 398.351 903 988 871 171 533 488 327

Table A2.

	Triangular lattice: surface	
	Resistive susceptibility	Conductive susceptibility
0	0	0
1	4	4
2	32	8
3	184	22
4	893.833 333 333 333 333 333 333 333 333 3	57.733 333 333 333 333 333 333 333 333 33
5	3901.25	151.280 952 380 952 380 952 380 952 381 0
6	16 157.498 484 848 484 848 484 848 484 84	409.606 671 106 671 106 671 106 671 106 7

Table A2. (Continued)

	Triangular lattice: surface	
	Resistive susceptibility	Conductive susceptibility
7	63 096.908 874 458 874 458 874 458 874 46	1117.401 425 939 042 038 113 245 543 589
8	238 973.038 988 282 988 496 504 029 759 2	3076.286 933 361 808 500 140 662 332 822
9	880 175.925 407 038 417 962 918 711 867 3	8562.044 930 924 548 299 423 933 993 926
10	3 149 620.603 810 431 263 888 861 371 444	23 793.501 796 410 133 879 714 243 912 10
11	11 151 999.686 668 856 364 007 604 573 19	66 886.127 174 150 499 563 572 499 967 80
12	38 573 440.178 832 585 306 633 096 088 92	187 495.973 256 172 063 804 530 153 742 6
13	132 029 098.991 439 916 163 089 182 957 1	527 465.396 427 220 979 695 308 769 486 0
14	447 702 023.965 050 189 295 309 415 056 2	1 494 566.882 772 229 360 552 919 264 508

Table A3.

	Square lattice: bulk	
	Resistive susceptibility	Conductive susceptibility
0	0	0
1	4	4
2	24	6
3	108	12
4	362	25
5	1220	48.419 047 619 047 619 047 619 047 619 05
6	3398	97.612 121 212 121 212 121 212 121 212 12
7	10 386.133 333 333 333 333 333 333 333 33	192.827 705 627 705 627 705 627 705 626 5
8	25 433.066 666 666 666 666 666 666 666 67	387.347 981 085 495 764 688 675 972 973 4
9	75 001.808 695 652 173 913 043 478 260 88	771.958 520 480 165 728 848 914 356 925 5
10	168 121.381 366 459 627 329 192 546 583 8	1544.554 340 213 251 870 670 443 482 322
11	486 607.811 741 134 041 274 293 728 715 2	3094.155 951 932 606 789 242 917 412 340
12	1 022 684.639 188 806 236 002 817 756 934	6185.155 887 271 728 842 777 335 016 676
13	2 952 107.909 031 635 276 032 187 112 141	12 474.239 276 663 825 550 204 419 599 14
14	5 732 738.214 363 900 600 818 482 904 895	24 463.187 201 509 117 287 153 443 209 80
15	17 524 104.726 523 379 660 195 424 330 74	51 534.582 659 466 355 727 410 758 623 39
16	29 042 930.048 678 894 257 588 025 162 57	93 860.104 391 869 374 337 403 875 427 56
17	103 369 859.280 767 454 335 273 924 961 8	217 968.319 311 522 190 441 900 030 841 7
18	134 495 347.901 969 049 824 504 279 378 8	353 956.918 551 650 827 851 452 930 797 6

Table A4.

	Square lattice: surface	
	Resistive susceptibility	Conductive susceptibility
0	0	0
1	3	3
2	14	3.5
3	57	6.333 333 333 333 333 333 333 333 333 333
4	177	12.25
5	563.5	22.366 666 666 666 666 666 666 666 666 67
6	1469.666 666 666 666 666 666 666 666 667	42.003 030 303 030 303 030 303 030 303 01

Table A4. (Continued)

	Square lattice: surface	
	Resistive susceptibility	Conductive susceptibility
7	4300.300 000 000 000 000 000 000 000 000	79.692 207 792 207 792 207 792 207 792 24
8	10 150.766 666 666 666 666 666 666 666 67	151.863 893 111 088 050 769 912 625 362 8
9	28 532.263 043 478 260 869 565 217 391 31	294.639 269 954 790 602 629 661 698 736 0
10	63 250.540 372 670 807 453 416 149 068 36	563.688 920 933 314 291 149 468 524 977 3
11	171 908.704 818 673 612 502 504 508 115 1	1106.329 625 622 273 414 089 468 767 414
12	368 068.794 449 580 539 309 493 287 120 0	2130.673 573 466 530 507 956 321 826 655
13	980 249.567 690 838 953 406 576 960 100 1	4237.264 057 964 399 432 799 000 878 873
14	1 992 822.804 206 078 526 220 020 477 726	8021.397 968 346 614 336 131 314 696 170
15	5 485 231.823 551 368 319 265 454 469 427	16 697.107 947 807 634 983 318 530 575 69
16	10 037 269.376 071 325 520 048 030 653 56	29 803.725 761 950 969 033 798 818 262 24
17	30 084 782.330 302 897 530 450 995 705 91	66 716.378 089 407 031 105 157 277 102 09
18	48 798 807.769 571 032 399 640 072 274 42	111 889.715 103 433 481 265 935 589 832 5

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